Calculation of load capacity of spur and helical gears —

Part 3: Calculation of tooth bending strength
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 6336-3 was prepared by Technical Committee ISO/TC 60, Gears, Subcommittee SC 2, Gear capacity calculation.

This second edition cancels and replaces the first edition (ISO 6336-3:1996), Clauses 5 and Clause 9 of which have been technically revised, with a new Clause 8 having been added to this new edition. It also incorporates the Technical Corrigendum ISO 6336-3:1996/Cor.1:1999.

ISO 6336 consists of the following parts, under the general title Calculation of load capacity of spur and helical gears:

— Part 1: Basic principles, introduction and general influence factors
— Part 2: Calculation of surface durability (pitting)
— Part 3: Calculation of tooth bending strength
— Part 5: Strength and quality of materials
— Part 6: Calculation of service life under variable load
Introduction

The maximum tensile stress at the tooth root (in the direction of the tooth height), which may not exceed the permissible bending stress for the material, is the basis for rating the bending strength of gear teeth. The stress occurs in the “tension fillets” of the working tooth flanks. If load-induced cracks are formed, the first of these often appears in the fillets where the compressive stress is generated, i.e. in the “compression fillets”, which are those of the non-working flanks. When the tooth loading is unidirectional and the teeth are of conventional shape, these cracks seldom propagate to failure. Crack propagation ending in failure is most likely to stem from cracks initiated in tension fillets.

The endurable tooth loading of teeth subjected to a reversal of loading during each revolution, such as “idler gears”, is less than the endurable unidirectional loading. The full range of stress in such circumstances is more than twice the tensile stress occurring in the root fillets of the loaded flanks. This is taken into consideration when determining permissible stresses (see ISO 6336-5).

When gear rims are thin and tooth spaces adjacent to the root surface narrow (conditions which can particularly apply to some internal gears), initial cracks commonly occur in the compression fillet. Since, in such circumstances, gear rims themselves can suffer fatigue breakage, special studies are necessary. See Clause 1.

Several methods for calculating the critical tooth root stress and evaluating some of the relevant factors have been approved. See ISO 6336-1.
Calculation of load capacity of spur and helical gears —

Part 3: Calculation of tooth bending strength

IMPORTANT — The user of this part of ISO 6336 is cautioned that when the method specified is used for large helix angles and large pressure angles, the calculated results should be confirmed by experience as by Method A.

1 Scope

This part of ISO 6336 specifies the fundamental formulae for use in tooth bending stress calculations for involute external or internal spur and helical gears with a rim thickness $s_R > 0.5 h_t$ for external gears and $s_R > 1.75 m_n$ for internal gears. In service, internal gears can experience failure modes other than tooth bending fatigue, i.e. fractures starting at the root diameter and progressing radially outward. This part of ISO 6336 does not provide adequate safety against failure modes other than tooth bending fatigue. All load influences on tooth stress are included in so far as they are the result of loads transmitted by the gears and in so far as they can be evaluated quantitatively.

The given formulae are valid for spur and helical gears with tooth profiles in accordance with the basic rack standardized in ISO 53. They may also be used for teeth conjugate to other basic racks if the virtual contact ratio $\varepsilon_{\alpha n}$ is less than 2.5.

The load capacity determined on the basis of permissible bending stress is termed “tooth bending strength”. The results are in good agreement with other methods for the range, as indicated in the scope of ISO 6336-1.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile


ISO 6336-1:2006, Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors

ISO 6336-5:2003, Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of material

3 Terms, definitions, symbols and abbreviated terms

For the purposes of this document, the terms, definitions, symbols and abbreviated terms given in ISO 1122-1 and ISO 6336-1 apply.
4 Tooth breakage and safety factors

Tooth breakage usually ends the service life of a transmission. Sometimes, the destruction of all gears in a transmission can be a consequence of the breakage of one tooth. In some instances, the transmission path between input and output shafts is broken. As a consequence, the chosen value of the safety factor $S_F$ against tooth breakage should be larger than the safety factor against pitting.

General comments on the choice of the minimum safety factor can be found in ISO 6336-1:2006, 4.1.7. It is recommended that manufacturer and customer agree on the value of the minimum safety factor.

This part of ISO 6336 does not apply at stress levels above those permissible for $10^3$ cycles, since stresses in this range may exceed the elastic limit of the gear tooth.

5 Basic formulae

The actual tooth root stress $\sigma_F$ and the permissible (tooth root) bending stress $\sigma_{FP}$ shall be calculated separately for pinion and wheel; $\sigma_F$ shall be less than $\sigma_{FP}$.

5.1 Safety factor for bending strength (safety against tooth breakage), $S_F$

Calculate $S_F$ separately for pinion and wheel:

$$S_{F1} = \frac{\sigma_{FG1}}{\sigma_{F1}} \geq S_{F\text{min}}$$  \hspace{1cm} (1)

$$S_{F2} = \frac{\sigma_{FG2}}{\sigma_{F2}} \geq S_{F\text{min}}$$  \hspace{1cm} (2)

$\sigma_{F1}$ and $\sigma_{F2}$ are derived from Equations (3) and (4). The values of $\sigma_{FG}$ for reference stress and static stress are calculated in accordance with 5.3.2.1 and 5.3.2.2, using Equation (5). For limited life, $\sigma_{FG}$ is determined in accordance with 5.3.3.

The values of tooth root stress limit $\sigma_{FG}$, of permissible stress $\sigma_{FP}$ and of tooth root stress $\sigma_F$ may each be determined by different methods. The method used for each value shall be stated in the calculation report.

NOTE Safety factors in accordance with the present clause are relevant to transmissible torque.

See ISO 6336-1:2006, 4.1.7 for comments on numerical values for the minimum safety factor and risk of damage.

5.2 Tooth root stress, $\sigma_F$

Tooth root stress $\sigma_F$ is the maximum tensile stress at the surface in the root.

5.2.1 Method A

In principle, the maximum tensile stress can be determined by any appropriate method (finite element analysis, integral equations, conformal mapping procedures or experimentally by strain measurement, etc.). In order to determine the maximum tooth root stress, the effects of load distribution over two or more engaging teeth and changes of stress with changes of meshing phase shall be taken into consideration.

Method A is only used in special cases and, because of the great effort involved, is only justifiable in such cases.
5.2.2 Method B

According to this part of ISO 6336, the local tooth root stress is determined as the product of nominal tooth root stress and a stress correction factor \(^1\).

This method involves the assumption that the determinant tooth root stress occurs with application of load at the outer point of single pair tooth contact of spur gears or of the virtual spur gears of helical gears. However, in the latter case, the “transverse load” shall be replaced by the “normal load”, applied over the facewidth of the actual gear of interest.

For gears having virtual contact ratios in the range \(2 \leq \varepsilon \mu \alpha < 2.5\), it is assumed that the determinant stress occurs with application of load at the inner point of triple pair tooth contact. In ISO 6336, this assumption is taken into consideration by the deep tooth factor, \(Y_{DT}\). In the case of helical gears, the factor, \(Y_{\beta}\), accounts for deviations from these assumptions.

Method B is suitable for general calculations and is also appropriate for computer programming and for the analysis of pulsator tests (with a given point of application of loading).

The total tangential load in the case of gear trains with multiple transmission paths (planetary gear trains, split-path gear trains) is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by inserting a mesh load factor, \(K_{\gamma}\), to follow \(K_A\) in Equation (3), in order to adjust as necessary the average load per mesh.

\[
\sigma_F = \sigma_{F0} \cdot K_A \cdot K_v \cdot K_{FB} \cdot K_{FA}
\]

where

- \(\sigma_{F0}\) is the nominal tooth root stress, which is the maximum local principal stress produced at the tooth root when an error-free gear pair is loaded by the static nominal torque and without any pre-stress such as shrink fitting, i.e. stress ratio \(R = 0\) [see Equation (4)];
- \(\sigma_{FP}\) is the permissible bending stress (see 5.3);
- \(K_A\) is the application factor (see ISO 6336-6), which takes into account load increments due to externally influenced variations of input or output torque;
- \(K_v\) is the dynamic factor (see ISO 6336-1), which takes into account load increments due to internal dynamic effects;
- \(K_{FB}\) is the face load factor for tooth root stress (see ISO 6336-1), which takes into account uneven distribution of load over the facewidth due to mesh-misalignment caused by inaccuracies in manufacture, elastic deformations, etc.;
- \(K_{FA}\) is the transverse load factor for tooth root stress (see ISO 6336-1), which takes into account uneven load distribution in the transverse direction, resulting, for example, from pitch deviations.

NOTE See ISO 6336-1:2006, 4.1.14, for the sequence in which factors \(K_A\), \(K_v\), \(K_{FB}\) and \(K_{FA}\) are calculated.

\[
\sigma_{F0} = \frac{F_1}{b_{mn}} \cdot Y_F \cdot Y_S \cdot Y_{FB} \cdot Y_B \cdot Y_{DT}
\]

\(^1\) Stresses such as those caused by the shrink-fitting of gear rims, which are superimposed on stresses due to tooth loading, should be taken into consideration in the calculation of permissible tooth root stress \(\sigma_{FP}\).
where

\( F_t \) is the nominal tangential load, the transverse load tangential to the reference cylinder \(^2\) (see ISO 6336-1);

\( b \) is the facewidth (for double helical gears \( b = 2 h_B \)) \(^3\);

\( m_n \) is the normal module;

\( Y_F \) is the form factor (see Clause 6), which takes into account the influence on nominal tooth root stress of the tooth form with load applied at the outer point of single pair tooth contact;

\( Y_S \) is the stress correction factor (see Clause 7), which takes into account the influence on nominal tooth root stress, determined for application of load at the outer point of single pair tooth contact, to the local tooth root stress, and thus, by means of which, are taken into account;

i) the stress amplifying effect of change of section at the tooth root, and

ii) the fact that evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented;

\( Y_\beta \) is the helix angle factor (see Clause 8), which compensates for the fact that the bending moment intensity at the tooth root of helical gears is, as a consequence of the oblique lines of contact, less than the corresponding values for the virtual spur gears used as bases for calculation;

\( Y_B \) is the rim thickness factor (see Clause 9), which adjusts the calculated tooth root stress for thin rimmed gears;

\( Y_{DT} \) is the deep tooth factor (see Clause 10), which adjusts the calculated tooth root stress for high precision gears with a contact ratio in the range of \( 2 < \varepsilon_{\alpha n} < 2.5 \).

5.3 Permissible bending stress, \( \sigma_{FP} \)

The limit value of tooth root stresses (see Clause 11) should preferably be derived from material tests using gears as test pieces, since in this way the effects of test piece geometry, such as the effect of the fillet at the tooth roots, are included in the results. The calculation methods provided constitute empirical means for comparing stresses in gears of different dimensions with experimental results. The closer test gears and test conditions resemble the service gears and service conditions, the lesser will be the influence of inaccuracies in the formulation of the calculation expressions.

5.3.1 Methods for determination of permissible bending stress, \( \sigma_{FP} \) — Principles, assumptions and application

Several procedures for the determination of permissible bending stress \( \sigma_{FP} \) are acceptable. The method adopted shall be validated by carrying out careful comparative studies of well-documented service histories of a number of gears.

\(^2\) In all cases, even when \( \varepsilon_{\alpha n} > 2 \), it is necessary to substitute the relevant total tangential load as \( F_t \). Reasons for the choice of load application at the reference cylinder are given in 6.3. See ISO 6336-1, 4.2, for definition of \( F_t \) and comments on particular characteristics of double helical gears.

\(^3\) The value \( b \), of mating gears, is the facewidth at the root circle, ignoring any intentional transverse chamfers or tooth-end rounding. If the facewidths of the pinion and wheel are not equal, it can be assumed that the load bearing width of the wider facewidth is equal to the smaller facewidth plus such extension of the wider that does not exceed \( 1 \times \) the module at each end of the teeth.
5.3.1.1 Method A

By this method, the values for \( \sigma_{FP} \) or for the tooth root stress limit, \( \sigma_{FG} \), are obtained using Equations (3) and (4) from the S-N curve or damage curve derived from results of testing facsimiles of the actual gear pair, under the appropriate service conditions.

The cost required for this method is, in general, only justifiable for the development of new products, failure of which would have serious consequences (e.g. for manned space flights).

Similarly, in line with this method, the allowable stress values may be derived from consideration of dimensions, service conditions and performance of carefully monitored reference gears.

5.3.1.2 Method B

Damage curves characterized by the nominal stress number (bending), \( \sigma_{F \text{lim}} \), and the factor \( Y_{NT} \) have been determined for a number of common gear materials and heat treatments from results of gear load or pulsator testing of standard reference test gears. Material values so determined are converted to suit the dimensions of the gears of interest, using the relative influence factors for notch sensitivity, \( Y_{\delta \text{rel T}} \), for surface roughness, \( Y_{R \text{rel T}} \), and for size, \( Y_X \).

Method B is recommended for the calculation of reasonably accurate gear ratings whenever bending strength values are available from gear tests, from special tests or, if the material is similar, from ISO 6336-5.

5.3.2 Permissible bending stress, \( \sigma_{FP} \): Method B

Subject to the reservations given in 5.3.2.1 and 5.3.2.2, Equation (5) is to be used for this calculation:

\[
\sigma_{FP} = \frac{\sigma_{F \text{lim}} Y_{ST} Y_{NT}}{S_{F \text{min}}} Y_{\delta \text{rel T}} Y_{R \text{rel T}} Y_X = \frac{\sigma_{FE} Y_{NT}}{S_{F \text{min}}} Y_{\delta \text{rel T}} Y_{R \text{rel T}} Y_X = \frac{\sigma_{FG}}{S_{F \text{min}}}
\]

where

\( \sigma_{F \text{lim}} \) is the nominal stress number (bending) from reference test gears (see ISO 6336-5), which is the bending stress limit value relevant to the influences of the material, the heat treatment and the surface roughness of the test gear fillets;

\( \sigma_{FE} \) is the allowable stress number for bending, corresponding to the basic bending strength of the un-notched test piece, under the assumption that the material condition (including heat treatment) is fully elastic

\[
\sigma_{FE} = (\sigma_{F \text{lim}} Y_{ST});
\]

\( Y_{ST} \) is the stress correction factor, relevant to the dimensions of the reference test gears (see 7.4);

\( Y_{NT} \) is the life factor for tooth root stress, relevant to the dimensions of the reference test gear (see Clause 12), which takes into account the higher load capacity for a limited number of load cycles;

\( \sigma_{FG} \) is the tooth root stress limit;

\[
\sigma_{FG} = (\sigma_{FP} S_{F \text{min}});
\]

\( S_{F \text{min}} \) is the minimum required safety factor for tooth root stress (see Clause 4 and 5.1);

\( Y_{\delta \text{rel T}} \) is the relative notch sensitivity factor, which is the quotient of the notch sensitivity factor of the gear of interest divided by the standard test gear factor (see Clause 13) and which enables the influence of the notch sensitivity of the material to be taken into account;