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Statistisk tolkning av data – Bestämning av statistiska toleransgränser

Statistical interpretation of data – Determination of a statistical tolerance interval

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SIS – Standardiseringskommissionen i Sverige

Standarden utarbetad av

SIS STANDARDISERINGSGRUPP

SVENSK STANDARD SS 01 42 12
ISO 3207

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Statistisk tolkning av data — Bestämning av statistiska toleransgränser

Denna standard utgörs av den engelska versionen av de internationella standarderna ISO 3207–1975, Statistical interpretation of data – Determination of a statistical tolerance interval och ISO 3207–1975/Addendum 1.

Standarden dokumenterar en viktig statistisk teknik. Den kan utgöra en referens om överenskommelse träffas att använda de i standarden ingående metoderna. Som exempel må nämnas i anvisningar för provning eller statistisk kontroll av processer och produkter. Den kan användas i utbildning, men utgör ingen lärobok. I standarden anges ej i vilka konkreta sammanhang den är tillämplig.

Den i denna standard angivna internationella standarden ISO 2854 gäller som svensk standard SS 01 42 11 – ISO 2854, Statistisk tolkning av data – Skattning och hypotestest avseende medelvärde och varians.

Se även SS 01 42 01, Statistik – Terminologi.

Statistical interpretation of data – Determination of a statistical tolerance interval

This standard consists of the English version of the international standards ISO 3207–1975, Statistical interpretation of data – Determination of a statistical tolerance interval and ISO 3207–1975/Addendum 1.

The standard documents an important statistical technique. It can be used as a reference in case of agreements where one considers that the methods in the standard ought to be used, for instance in instructions for testing or statistical inspection of processes and products. It can be used in education, but is no textbook. No specific situation where the standard can be applied is stated in it.

The international standard, ISO 2854, mentioned in this standard is valid as Swedish standard SS 01 42 11 – ISO 2854, Statistical interpretation of data – Techniques of estimation and tests relating to means and variances.

See also SS 01 42 01, Statistics – Terminology.

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Prefixet SS införs som beteckning för svensk standard utgiven 1978-01-01 och senare. Vid revidering av äldre standarder ersätter SS prefixen SEN, SIS och SMS. Den numeriska delen behålls i regel oförändrad. Standard utarbetad av SEK får 7-siffrig numerisk del med siffran 4 före de klassificerande sex siffrorna.

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Statistical interpretation of data — Determination of a statistical tolerance interval

SECTION ONE : FORMAL PRESENTATION OF RESULTS

GENERAL REMARKS

1) This International Standard specifies methods enabling a sample to be used as the basis for determining a statistical tolerance interval, i.e. an interval such that there is a fixed probability (confidence level) that the interval will contain at least a proportion p of the population from which the sample is taken. The statistical tolerance interval may be two-sided or one-sided. The limits of the interval are called "statistical tolerance limits"; they are also called "natural limits of the process".

2) These methods are applicable only where it may be assumed that in the population under consideration the sample units have been selected at random and are independent.

3) The methods described below apply also only on condition that the distribution of the characteristic being studied is normal. The requirement of normality is more important here than for the inferences on means and differences between means in ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

4) In order to check the hypothesis of normality, the methods laid down in ISO. . ., *Statistical interpretation of data — Normality tests*[1], are used.

5) Where the hypothesis of normality has to be rejected or where there is some reason to doubt its validity, one may envisage transforming the variate to make it normal or applying the method described in the introductory remark of annex A of this International Standard.

It is also possible to apply now methods which allow the determination of statistical tolerance intervals for other distribution forms than normal distributions. The description of these methods has not been considered in this International Standard.

6) In determining a statistical tolerance interval, it is desirable in connection with the origin or the method of collection of data to give all information that may assist in their statistical analysis, in particular the smallest unit or fraction of a measurement unit having practical significance.

7) No elimination or potential correction of individual data that are doubtful shall be carried out unless there are experimental, technical or obvious reasons to provide circumstantial justification of such elimination or correction.

In every instance, mention shall be made of the data eliminated or corrected.

8) As stated in 1), the confidence level $1 - \alpha$ is the probability that the statistical tolerance interval will contain at least a proportion p of the population. The risk of this interval containing less than a proportion p of the population is α. The most usual values of $1 - \alpha$ are 0,95 and 0,99 ($\alpha = 0{,}05$ and 0,01).

This means that if statistical tolerance intervals are determined for a large number of samples at the confidence level 0,95 for example, the proportion of those intervals which will contain at least the desired fraction of the population will be close to 95 %.

9) Tables 1 and 2 are applicable to the case where the standard deviation for the population is known (the mean being unknown); tables 3 and 4 to the case where the mean and the standard deviation are unknown.

Where the mean and the standard deviation having respectively the values m and σ are known, the distribution of the characteristic under investigation (assumed to be normal) is fully determined; there is exactly a proportion p of the population :

- on the right side of $m - u_p \sigma$ } one-sided intervals
- on the left side of $m + u_p \sigma$ } one-sided intervals
- between $m - u_{(1+p)/2}\, \sigma$ and $m + u_{(1+p)/2}\, \sigma$: two-sided interval

where u_p is the fractile of order p of the standardized normal variate.

Numerical values of u_p are in these cases to be read on the bottom line of tables 5 and 6.

10) The calculations can often be very much simplified by making a change in origin and/or in unit.

1) In preparation.

TABLE 1 — **One-sided statistical tolerance interval** (known variance)[1]

Technical characteristics of the population under investigation[2] . . . .

Technical characteristics of the sample units[2] . . . .

Eliminated observations[3] . . . .

Statistical data	Calculations
Sample size : $n =$	
Sum of the observed values : $\Sigma x =$	$\bar{x} = \frac{\Sigma x}{n} =$
Known value of the variance of the population : $\sigma^2 =$	
whence the standard deviation : $\sigma =$	$k_1\ (n, p, 1-\alpha)\ \sigma =$ [6]
Proportion of the population selected for the statistical tolerance interval[4] : $p =$	
Chosen confidence level[5] : $1-\alpha =$	
$k_1\ (n, p, 1-\alpha) =$	

Results

a) One-sided interval "to the left"

There is a probability $1-\alpha$ that at least a proportion p of the population is above :

$$L_s = \bar{x} + k_1\ (n, p, 1-\alpha)\ \sigma =$$

b) One-sided interval "to the right"

There is a probability $1-\alpha$ that at least a proportion p of the population is above :

$$L_i = \bar{x} - k_1\ (n, p, 1-\alpha)\ \sigma =$$

1) A numerical example is given in section two of this International Standard : example No. 1
2) See paragraph 6 of General remarks.
3) See paragraph 7 of General remarks.
4) See paragraph 1 of General remarks.
5) See paragraph 8 of General remarks.
6) The values of $k_1\ (n, p, 1-\alpha)$ can be read directly from table 5 for different values of n, and for

$$p = 0{,}90;\ 0{,}95;\ 0{,}99$$

$$1-\alpha = 0{,}95 \text{ and } 0{,}99$$

TABLE 2 — **Two-sided statistical tolerance interval** (known variance)[1]

Technical characteristics of the population under investigation[2] . . . . Technical characteristics of the sample units[2] . . . . Eliminated observations[3]	
Statistical data Sample size : $n =$ Sum of the observed values : $\Sigma x =$ Known value of the variance of the population : $\sigma^2 =$ whence the standard deviation : $\sigma =$ Proportion of the population selected for the statistical tolerance interval[4] : $p =$ Chosen confidence level[5] : $1 - \alpha =$ $k'_1\ (n, p, 1 - \alpha) =$	**Calculations** $\overline{x} = \frac{\Sigma x}{n} =$ $k'_1\ (n, p, 1 - \alpha)\ \sigma =$ [6]
Results There is a probability $1 - \alpha$ that at least a proportion p of the population is included between the limits[7] : $L_i = \overline{x} - k'_1\ (n, p, 1 - \alpha)\ \sigma =$ $L_s = \overline{x} + k'_1\ (n, p, 1 - \alpha)\ \sigma =$	

1) A numerical example is given in section two of this International Standard : example No. 2.
2) See paragraph 6 of General remarks.
3) See paragraph 7 of General remarks.
4) See paragraph 1 of General remarks.
5) See paragraph 8 of General remarks.
6) The values of $k'_1\ (n, p, 1 - \alpha)$ can be read from table 6 for different values of n, and for

$$p = 0{,}90;\ 0{,}95;\ 0{,}99$$

$$1 - \alpha = 0{,}95 \text{ and } 0{,}99$$

7) These limits are symmetrical about $\overline{x}$ but they are not "symmetrical in probability". It is not true that at the confidence level $1 - \alpha$, a proportion not exceeding $(1 - p)/2$ of the population is below L_i and a proportion not exceeding $(1 - p)/2$ is above L_s.

TABLE 3 — **One-sided statistical tolerance interval** (unknown variance)[1)]

Technical characteristics of the population under investigation[2)] . . . .

Technical characteristics of the sample units[2)] . . . .

Eliminated observations[3)] . . . .

Statistical data	**Calculations**
Sample size : $n =$	$\bar{x} = \frac{\Sigma x}{n} =$
Sum of the observed values : $\Sigma x =$	$\frac{\Sigma (x - \bar{x})^2}{n-1} = \frac{\Sigma x^2 - (\Sigma x)^2/n}{n-1} =$
Sum of the squares of the observed values : $\Sigma x^2 =$	
Proportion of the population selected for the statistical tolerance interval[4)] : $p =$	$\sigma^* = s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}} =$ (estimation of the standard deviation σ)
Chosen confidence level[5)] : $1 - \alpha =$	
$k_2 (n, p, 1-\alpha) =$	$k_2 (n, p, 1-\alpha)\, s =$ [6)]

Results

a) One-sided interval "to the left"

There is a probability $1 - \alpha$ that at least a proportion p of the population is below :

$$L_s = \bar{x} + k_2 (n, p, 1-\alpha)\, s =$$

b) One-sided interval "to the right"

There is a probability $1 - \alpha$ that at least a proportion p of the population is above :

$$L_i = \bar{x} - k_2 (n, p, 1-\alpha)\, s =$$

1) A numerical example is given in section two of this International Standard : example No. 3.
2) See paragraph 6 of General remarks.
3) See paragraph 7 of General remarks.
4) See paragraph 1 of General remarks.
5) See paragraph 8 of General remarks.
6) The values of k_2 $(n, p, 1-\alpha)$ can be read from table 7 for different values of n, and for

$$p = 0{,}90;\ 0{,}95;\ 0{,}99$$

$$1 - \alpha = 0{,}95 \text{ and } 0{,}99$$