SWEDISH STANDARDS INSTITUTE

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Utgåva 1

Statistisk tolkning av data – Skattning och hypotestest avseende medel

Statistical interpretation of data – Techniques of estimation and tests relating to means and variances

Efter översyn av rubricerad svenska standard har ansvarig SIS/TK beslutat att det tekniska innehållet i standarden skall fortsätta att gälla som svensk standard.

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Om det råder oklarhet i något avseende huruvida bekräftad äldre svensk standard bör eller kan tillämpas i en situation kan hänvändelse ske till det verksamhetsområde (SIS/VO) som handlägger standarden.

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SIS - Standardiseringskommissionen i Sverige

Standarden utarbetad av

SIS STANDARDISERINGSGRUPP

SVENSK STANDARD SS 01 42 11
ISO 2854

Första giltighetsdag 1979-07-01 | Utgåva 1 | Sida 1 (48)

Statistisk tolkning av data — Skattning och hypotestest avseende medelvärde och varians

Denna standard utgörs av den engelska versionen av den internationella standarden ISO 2854—1976, Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.

Standarden dokumenterar en viktig statistisk teknik. Den kan utgöra en referens om överenskommelse träffas att använda de i standarden ingående metoderna. Som exempel må nämnas i anvisningar för provning eller statistisk kontroll av processer och produkter. Den kan användas i utbildning, men utgör ingen lärobok. I standarden anges ej i vilka konkreta sammanhang den är tillämplig.

De i denna standard angivna ISO-standarderna gäller som svensk standard enligt följande:

ISO 2602 = SS 01 42 10—ISO 2602 — Statistisk tolkning av data — Skattning av medelvärde — Konfidensintervall.

ISO 3207 = SS 01 42 12—ISO 3207 — Statistisk tolkning av data — Bestämning av statistiska toleransgränser.

ISO 3301 = SS 01 42 13—ISO 3301 — Statistisk tolkning av data — Jämförelse av två medelvärden erhållna vid parvisa observationer.

Se även SS 01 42 01 — Statistik — Terminologi.

Statistical interpretation of data — Techniques of estimation and tests relating to means and variances

This standard consists of the English version of the international standard ISO 2854—1976, Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.

The standard documents an important statistical technique. It can be used as a reference in case of agreements where one considers that the methods in the standard ought to be used, for instance in instructions for testing or statistical inspection of processes and products. It can be used in education, but is no textbook. No specific situation where the standard can be applied is stated in it.

Those in this standard mentioned international standards are valid as Swedish standards according to the following:

ISO 2602 = SS 01 42 10—ISO 26 02 — Statistical interpretation of data — Estimation of the mean — Confidence interval.

ISO 3207 = SS 01 42 12—ISO 3207 — Statistical interpretation of data — Determination of a statistical tolerance interval.

ISO 3301 = SS 01 42 13—ISO 3301 — Statistical interpretation of data — Comparison of two means in the case of paired observations.

Se also SS 01 42 01 — Statistics — Terminology.

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UDK 519.28

Prefixet SS införs som beteckning för svensk standard utgiven 1978-01-01 och senare. Vid revidering av äldre standarder ersätter SS prefixen SEN, SIS och SMS. Den numeriska delen behålls i regel oförändrad. Standard utarbetad av SEK får 7-siffrig numerisk del med siffran 4 före de klassificerande sex siffrorna.

CONTENTS

MEANS	Variance known	Variance unknown
Comparison of a mean with a given value	A	A′
Estimation of a mean	B	B′
Comparison of two means	C	C′
Estimation of the difference of two means	D	D′

VARIANCES	
Comparison of a variance with a given value	E
Estimation of a variance	F
Comparison of two variances	G
Estimation of the ratio of two variances	H

Statistical interpretation of data – Techniques of estimation and tests relating to means and variances

SECTION ONE : PRESENTATION OF CALCULATIONS

GENERAL REMARKS

1) This International Standard specifies the techniques required :

a) to estimate the mean or the variance of populations;

b) to examine certain hypotheses concerning the value of those parameters, from samples.

2) The techniques used are valid only if, in each of the populations under consideration, the sample elements are drawn at random and are independent. In the case of a finite population, elements drawn at random may be considered as independent when the population size is sufficiently large or when the sampling fraction is sufficiently small (for instance smaller than 1/10).

3) The distribution of the observed variable is assumed to be normal in each population. However, if the distribution does not deviate very much from the normal, the techniques described remain approximately valid to an extent sufficient for most practical applications, provided the sample size is not too small. For tables A, B, C and D, the sample size should be of the order of 5 to 10 at least; for all the other tables, it should be not less than about 20.[1]

4) A certain number of techniques exist which permit the verification of the hypothesis of normality. This subject is dealt with briefly in the examples in section two and will also be dealt with in a further document (yet to be prepared). Nevertheless, this hypothesis may be admitted on the basis of information other than that provided by the sample itself. In the case where the hypothesis of normality should be rejected, the obvious method to follow is to resort to non-parametric tests or to use suitable transformations for obtaining normally distributed populations, for example $1/x$, $\log(x + a)$, $\sqrt{x + a}$, but the conclusions reached by applying these procedures described in this International Standard are only directly valid for the transformed variate; caution should be used in the translation to the original variate. For example **exp (mean log** x**)** is equal to the **geometric mean** of x not the arithmetic mean.

If what is really needed is an estimate of the mean or standard deviation of the variate X itself then, whether the population distribution is normal or not, an unbiassed estimation of the mean m and the population variance σ^2 is produced by the sample mean $\bar{x}$ and characteristic s^2.

5) It is desirable to accompany each statistical operation with all the particulars relevant to the source or to the method of obtaining the observations which may clarify this statistical analysis, and in particular to give the unit or the smallest unit of measurement having practical meaning.

6) It is not permissible to discard any observations or to apply any corrections to apparently doubtful observations without a justification based on experimental, technical or other evident grounds which should be clearly given. In any case the discarded or corrected values and the reason for discarding or correcting them must be mentioned.

7) In problems of estimation, the confidence level $1 - \alpha$ is the probability that the confidence interval covers the true value of the estimated parameter. Its most usual values are 0,95 and 0,99, or $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

8) In problems of testing a hypothesis, the significance level is, in the two-sided cases, the probability of rejecting the null hypothesis (or tested hypothesis) if it is true (error of the first kind); in the one-sided cases, the significance level is the maximum value of this probability (maximum value of the error of the first kind). Values of $\alpha = 0{,}05$ (1 in 20 chance) or 0,01 (1 in 100 chance) are very commonly employed according to the risk which the user is prepared to take. Since a hypothesis may be rejected using $\alpha = 0{,}05$, but not when using 0,01, it is often appropriate to use the phrase : "the hypothesis is rejected at the 5 % level" or, if this is the case, "at the 1 % level". Attention is drawn to the existence of an error of the second kind. This error is committed if the null hypothesis is accepted when it is false. Terms concerning statistical tests are defined in clause 2 of ISO 3534, *Statistics – Vocabulary*[2].

1) Studies about normal distributions are in progress in TC 69/SC 2.

2) At present at the stage of draft.

9) The calculations can often be greatly reduced by making a change of origin and/or unit on the data. In the case of data classified into groups, reference may be made to the formulae in ISO 2602, *Statistical interpretation of test results — Estimation of the mean — Confidence interval.*

NOTE — A change of origin may be essential to obtain sufficient accuracy when calculating a variance using the stated formulae with a low precision calculator or computer.

10) The methods shown in tables C and C′ deal with the comparison of two means. They assume that the corresponding samples are independent. For the study of certain problems, it may be interesting to pair the observations (for instance in the comparison of two methods or the comparison of two instruments). The statistical treatment of paired observations is the subject of ISO 3301, *Statistical interpretation of data — Comparison of two means in the case of paired observations*, but in annex A an example of treatment of paired observations is given. It uses formally the data of table A′.

11) The symbols and their definitions used in this International Standard are in conformity with ISO 3207, *Statistical interpretation of data — Determination of a statistical tolerance interval.*

TABLES

TABLE A — **Comparison of a mean with a given value** (variance known)

Technical characteristics of the population studied (5) . . . . Technical characteristics of the sample items (5) . . . . Discarded observations (6)	
Statistical data Sample size : $n =$ Sum of the observed values : $\Sigma x =$ Given value : $m_0 =$ Known value of the population variance : $\sigma^2 =$ Or standard deviation : $\sigma =$ Significance level chosen (8) : $\alpha =$	**Calculations** $\bar{x} = \frac{\Sigma x}{n} =$ $[u_{1-\alpha}/\sqrt{n}\,]\,\sigma =$ $[u_{1-\alpha/2}/\sqrt{n}]\,\sigma =$

Results

Comparison of the population mean with the given value m_0 :

Two-sided case :

The hypothesis of the equality of the population mean to the given value (null hypothesis) is rejected if :

$$|\bar{x} - m_0| > [u_{1-\alpha/2}/\sqrt{n}\,]\,\sigma$$

One-sided cases :

a) The hypothesis that the population mean is not smaller than m_0 (null hypothesis) is rejected if :

$$\bar{x} < m_0 - [u_{1-\alpha}/\sqrt{n}\,]\,\sigma$$

b) The hypothesis that the population mean is not greater than m_0 (null hypothesis) is rejected if :

$$\bar{x} > m_0 + [u_{1-\alpha}/\sqrt{n}\,]\,\sigma$$

NOTE — The numbers (5), (6) and (8) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The significance level α (see § 8 of the "General remarks") is the probability of rejecting the null hypothesis when this hypothesis is true.

2) U stands for the standardized normal variate : the value u_α is defined by :

$$P\,[U < u_\alpha] = \alpha$$

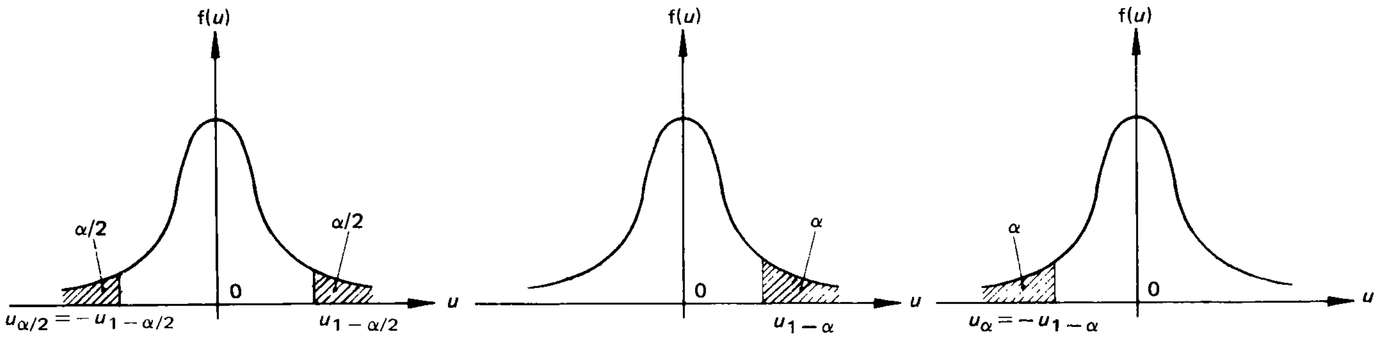
Since the distribution of U is symmetrical around zero, $u_\alpha = -u_{1-\alpha}$.

We therefore have :

$$P\,[U > u_\alpha] = 1 - \alpha$$

$$P\,[-u_{1-\alpha/2} < U < u_{1-\alpha/2}] = 1 - \alpha$$

Probability density of U (standardized normal distribution)



Two-sided case One-sided cases

3) $\sigma/\sqrt{n}$ is the standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $u_{1-\alpha}/\sqrt{n}$ and $u_{1-\alpha/2}/\sqrt{n}$ are given in table 1 of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

EXAMPLE : see section two, "Explanatory notes and examples".

TABLE A' – **Comparison of a mean with a given value** (variance unknown)

Technical characteristics of the population studied (5) . . . .

Technical characteristics of the sample items (5) . . . .

Discarded observations (6) . . . .

Statistical data	**Calculations**
Sample size :	$\bar{x} = \frac{\Sigma x}{n} =$
$n =$	
Sum of the observed values :	$\frac{\Sigma (x-\bar{x})^2}{n-1} = \frac{\Sigma x^2 - (\Sigma x)^2/n}{n-1}$
$\Sigma x =$	
Sum of the squares of the observed values :	$\sigma^* = s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}} =$
$\Sigma x^2 =$	
Given value :	$[t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s =$
$m_0 =$	
Degrees of freedom :	
$\nu = n - 1$	
Significance level chosen (8) :	$[t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\, s =$
$\alpha =$	

Results

Comparison of the population mean with the given value m_0 :

Two-sided case :

The hypothesis of the equality of the population mean to the given value (null hypothesis) is rejected if :

$$|\bar{x} - m_0| > [t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\, s$$

One-sided cases :

a) The hypothesis that the population mean is not smaller than m_0 (null hypothesis) is rejected if :

$$\bar{x} < m_0 - [t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s$$

b) The hypothesis that the population mean is not greater than m_0 (null hypothesis) is rejected if :

$$\bar{x} > m_0 + [t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s$$

NOTE – The numbers (5), (6) and (8) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The significance level α (see § 8 of the "General remarks") is the probability of rejecting the null hypothesis when this hypothesis is true.

2) $t(\nu)$ stands for Student's variate with $\nu = n - 1$ degrees of freedom : the value $t_\alpha(\nu)$ is defined by

$$P\,[t(\nu) < t_\alpha(\nu)] = \alpha$$

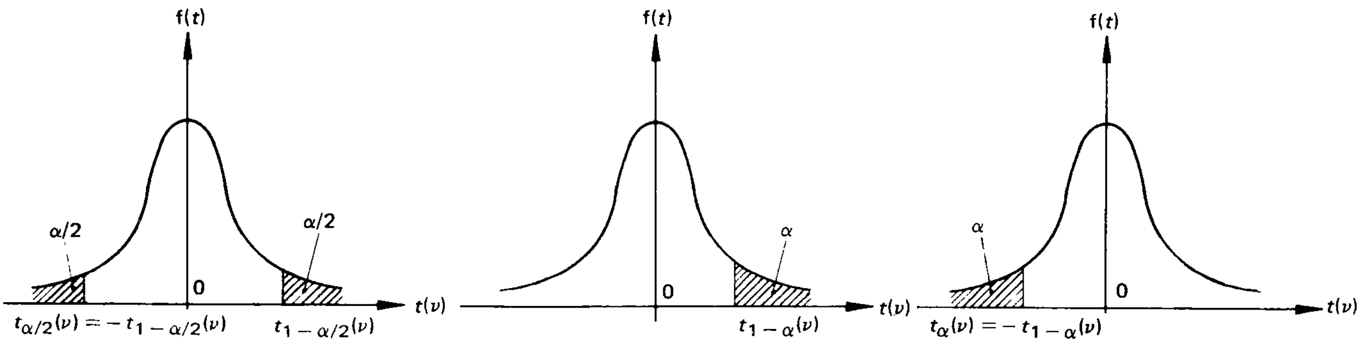
Since the distribution of $t(\nu)$ is symmetrical around zero, $t_\alpha(\nu) = -t_{1-\alpha}(\nu)$.

We therefore have :

$$P\,[t(\nu) > t_\alpha(\nu)] = 1 - \alpha$$

$$P\,[-t_{1-\alpha/2}(\nu) < t(\nu) < t_{1-\alpha/2}(\nu)] = 1 - \alpha$$

Probability density of Student's $t(\nu)$ with $\nu = n - 1$ degrees of freedom



Two-sided case

One-sided cases

3) $\sigma^*/\sqrt{n}$ is the estimated standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $t_{1-\alpha/2}(\nu)/\sqrt{n}$ and $t_{1-\alpha}(\nu)/\sqrt{n}$ are given in table IIb of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$

EXAMPLE : see section two, "Explanatory notes and examples".